Week 1

Matrices and Matrix Arithmetic

MATH2400 | Ryann Joseph | ryannjoseph.com | Fall 2025

1.1: Matrices

Types of Matrices

Zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(any matrix will all 0 entries)

Column matrix

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(any $m \times 1$ matrix)

Row matrix

$$\begin{bmatrix} 1 & 5 & 6 \end{bmatrix}$$

(any $1 \times m$ matrix)

Diagonal matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

(any matrix with all non-zero entries diagonal)

Identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(any square matrix will 1's along diagonal)

Upper triangular matrix

$$\begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$$

(any matrix with all non-zero entries on/above diagonal)

Lower triangular matrix

$$\begin{bmatrix} 1 & 0 \\ 3 & 9 \end{bmatrix}$$

(any matrix with all non-zero entries on/below diagonal)

Symmetric matrix

$$\begin{bmatrix} 1 & 5 \\ 5 & -1 \end{bmatrix}$$

(any square matrix where $A^T = A$)

Skew symmetric matrix

$$\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

(any square matrix where $A^T = -A$)
(therefore all diagonal entries must $= 0$)

Matrix convention

row x column

Transpose

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

1.1.1: Transpose Example

Find the transpose of the following matrix:

$$\begin{bmatrix} 1 & 2 & 6 \\ 3 & 0 & 1 \end{bmatrix}$$

1.1.2: Trace of Skew Symmetric Example

Describe the trace of a skew symmetric matrix

1.2: Matrix Arithmetic

Matrix Algebra	
Addition/Subtraction	• Must be same size matrices • Distributive • Commutative
Scalar Multiplication	• Distribute scalar to every entry of matrix • $cA = B$
Multiplication	• If A is m x k and B is k x n, *AB* is an m x n matrix • Associative • NOT commutative (order matters)

1.2.1: Matrix Product Example

Given the following matrices:

$$A = \begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine if each of the following matrix products exist, and if so, find their product.

$$AB, AC, BA, BC, CA, CB$$

1.2.2: Matrix Principles Example

Compute the following matrix from the matrices above:

$$(A + B)C$$

Worksheet questions

1. Show that the trace function on square matrices is *additive*, that is for two $n \times n$ matrices A and B, we have $\operatorname{tr}(A+B) = \operatorname{tr} A + \operatorname{tr} B$.

2. Recall that a square matrix A is called *symmetric* if $A^T = A$. We say that a square matrix A is *skew symmetric* (or *antisymmetric*) if $A^T = -A$.

Consider a skew symmetric matrix B with only partially known entries:

$$B = \begin{bmatrix} 0 & \boxed{?} & 3 & -2 \\ -2 & \boxed{?} & -1 & 1 \\ \boxed{?} & \boxed{?} & \boxed{?} & \boxed{?} \\ \boxed{?} & \boxed{?} & 2 & \boxed{?} \end{bmatrix}$$

a) What is b_{41}?

b) Recover the rest of the entries.

c) Compute $\operatorname{tr} B$.

Worksheet questions

3. For the matrices

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}$$

Determine which of the products AB and/or BA exist. If one fails to exist, explain why, and calculate the one(s) that do(es).

4. Let M be the matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

Calculate M^2, M^3 and M^4. Try to discern the pattern, and use what you conclude to predict M^5 without computing it.

What is M^n for any positive integer n?