11 Systems of differential equations

11.1 Basics of systems of differential equations

Convert to an equivalent first order system.

$$y'' + 17y' + 3y = e^x$$

$$X_{1} = y_{1}$$

$$Y_{2} = y_{1}$$

$$Y_{1} = x_{2}$$

$$Y_{1} + 17y_{1} + 3y_{2} = e^{x}$$

$$X_{2} + 17x_{2} + 3x_{1} = e^{x}$$

$$Y_{2} = -17x_{2} - 3x_{1} + e^{x}$$

$$X_{1} = 0$$

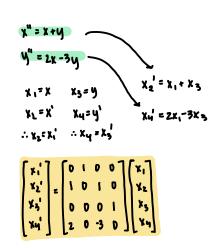
$$X_{2} = -17x_{2} - 3x_{1} + e^{x}$$

$$X_{1} = 0$$

$$X_{2} = -17x_{2} - 3x_{1} + e^{x}$$

Convert to an equivalent first order system.

$$x'' = x + y$$
$$y'' = 2x - 3y$$



Rewrite the following first-order system into a second-order equation for x(t).

$$x' = 3x + 2y$$

$$y' = x - y$$

$$x' = 3x + 2y \Rightarrow y = \frac{1}{2}x' - \frac{3}{2}x$$

$$x'' = 3x' + 2y' \Rightarrow y' = \frac{1}{2}x'' - \frac{3}{2}x'$$

$$\frac{1}{2}x'' - \frac{3}{2}x' = x - (\frac{1}{2}x' - \frac{3}{2}x)$$

$$\frac{1}{2}x'' - x' - \frac{5}{2}x = 0$$

11.2 Constant coefficient systems

Find the general and specific solutions of the following system.

$$x' = 2x - 3y$$
$$y' = -3x + 10y$$

$$x(0) = 1$$
 $y(0) = 4$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$R-A\Sigma = \begin{vmatrix} 2-A & -3 \\ -3 & 10-A \end{vmatrix} = \begin{pmatrix} A^{2}-12A+11 \\ -3 & 10-A \end{vmatrix}$$

$$= (A-11)(A-1)$$

$$A=1, 11$$

$$\begin{bmatrix} A=1 \\ 3 \\ 1 \end{bmatrix} = C_{1}e^{\frac{1}{3}} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = C_{1}e^{\frac{1}{3}} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + C_{2}e^{\frac{11}{3}} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix} = C_1 e^{0} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + C_2 e^{0} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$-1 = 3c_1 + c_2$$

$$4 = c_1 - 3c_2$$

$$c_1 = \frac{1}{10} \quad C_2 = \frac{15}{10}$$

$$\begin{bmatrix} y \\ y \end{bmatrix} = \frac{1}{10} e^{0} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{13}{10} e^{0} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Find the general and specific solutions of the following equation.
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$A-\Lambda I = \begin{vmatrix} -A & 1 \\ -1 & -A \end{vmatrix} = A^{2}+1=0$$

$$A=i$$

$$A-i I = \begin{bmatrix} -i & 1 & | & 0 \\ 1 & -i & | & 0 \end{bmatrix}$$

$$\begin{cases} i \\ i \end{bmatrix}$$

$$P(V) \quad India \quad e^{ill} \left((os(bl)+isin(bl)) \left[V \right]$$

$$= e^{ol} \left((os(l)+isin(l)) \left[i \right]$$

$$= \begin{bmatrix} cos(l)+isin(l) \\ i(os(l)+-in(l)) \end{bmatrix}$$

$$= \begin{bmatrix} cos(l)+isin(l) \\ i(os(l)+-in(l)) \end{bmatrix}$$

$$= \begin{bmatrix} cos(l)+isin(l) \\ i(os(l)+-in(l)) \end{bmatrix}$$

$$\begin{cases} v \\ -sin(l) \end{bmatrix} + i \begin{bmatrix} sin(l) \\ cos(l) \end{bmatrix}$$

$$\begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0$$