1 Matrices and matrix arithmetic

1.1 Matrices

Find the transpose of the following matrix:

$$\begin{bmatrix} 1 & 2 & 6 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 6 \\ 3 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 6 & 1 \end{bmatrix}$$

Describe the trace of a skew symmetric matrix

$A^T = -A$, diagonal of matrix does not change with ^{T}, $\therefore tr(A) = \sum 0 + 0 + \ldots + 0 = 0$

1.2 Matrix arithmetic

Given the following matrices:

$$A = \begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$$

Determine if each of the following matrix products exist, and if so, find their product.

$$AB, AC, BA, BC, CA, CB$$

AB

$\begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}$

$2\times3 \times 2\times3$

DNE

AC

$\begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$

$2\times3 \times 3\times2 = 2\times2$

$AC = \begin{bmatrix} 25 & 19 \\ 3 & 2 \end{bmatrix}$

BA

$\begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}\begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}$

$2\times3 \times 2\times3$

DNE

BC

$\begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$

$2\times3 \times 3\times2 = 2\times2$

$BC = \begin{bmatrix} 3 & 3 \\ 24 & 16 \end{bmatrix}$

CA

$\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix}$

$3\times2 \times 2\times3 = 3\times3$

$CA = \begin{bmatrix} 10 & 12 & 25 \\ 14 & 16 & 34 \\ 1 & 0 & 1 \end{bmatrix}$

CB

$\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}$

$3\times2 \times 2\times3 = 3\times3$

$CB = \begin{bmatrix} 7 & 3 & 12 \\ 17 & 6 & 20 \\ 4 & 3 & 6 \end{bmatrix}$

Compute the following matrix from the matrices above:

$$(A+B)C$$

$$A+B = \begin{bmatrix} 3 & 4 & 8 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 5 & 3 & 7 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 4 & 4 & 10 \\ 5 & 3 & 7 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 28 & 22 \\ 27 & 19 \end{bmatrix}$$