10 Physical applications and Cauchy-Euler equations

10.1 Springs and oscillations

We can use second order differential equations to model the behavior of the oscillations of spring systems. The general equation modeling the motion of a spring is:

$$my" + by' + my = F(t)$$

The entire equation is equal to the force of the system, where y is position

$my"$: **mass** times $y"$, or acceleration

by': **friction** force, where y' is velocity

ky: spring behavior, k is the **spring constant** and y is position

$F(t)$: a forcing term that can be $= 0$ or $\neq 0$ (an external force pushing on the spring).

<u>Damping</u>

When there is a friction term, $b \neq 0$, the system is exhibiting some sort of **damping**, which controls spring oscillations and energy dissipation. The type of damping can be three different types, based on the terms m, b, k.

b²□4km	**Type of damping**
$>$	Overdamped
$=$	Critically damped
$<$	Underdamped

The type of damping controls the end behavior of the system, where underdamped systems oscillate more than critically damped and overdamped systems. Damping reduces the oscillation of a spring system, so more damping results in less spring oscillations.

<u>Forced oscillations</u>

When there is a non-homogeneous term, $F(t) \neq 0$, the system is exhibiting a specific type of forcing, either transience, modulation, or resonance. We can use the frequency of the homogeneous system to determine this. The frequency, ω, is $\omega = \sqrt{\frac{k}{m}}$. The forcing term, $F(t)$, is always a sinusoid, usually either a $Fcos(\alpha t)$ term or a $Fsin(\alpha t)$ term.

Condition	**Type of forcing**
$b \neq 0$	Transience
$\omega = \alpha$	Resonance
$\omega \neq \alpha$	Modulation

As we can see from the table, if there is a forcing term, $F(t)$, and there is a damping term, it is always transience. If $b = 0$, then the determination between modulation and resonance must be done. ω must be calculated, and if that is equal to the frequency of the forcing term, α, then the system exhibits resonance, but if they are not equal, the system exhibits modulation.

You do not need to be able to solve spring systems, just be able to describe the system's damping & forcing.