1 Matrices and matrix arithmetic

1.1 Matrices

This course utilizes matrices, which are simply, down to it, boxes of different sizes with numbers in it. Matrices have uses in many different applications including linear algebra and outside of it. Computers use matrices to store pixels used on your screens, computer scientists use them for computations of extremely large datasets, and so many more. In this course, we use matrices to represent the elements in systems of linear equations and linear transformations. These are powerful tools to store the information about these linear algebra concepts.

Matrix Conventions

In this course, a matrix is a 2D figure, where each horizontal segment is called a **row** and each vertical segment is a **column**. We refer to the size of a matrix as its number of rows **times** its number of columns. Matrices can have more rows than columns, more columns than rows, or the same size. In this case, a matrix is called a **square matrix**.

row x column

2x3 matrix

$$\begin{bmatrix} 1 & 6 & 5 \\ 8 & 1 & 4 \end{bmatrix}$$

4x2 matrix

$$\begin{bmatrix} 5 & 3 \\ 1 & 4 \\ 6 & 9 \\ -7 & 2 \end{bmatrix}$$

3x3 matrix
square matrix

$$\begin{bmatrix} 5 & 3 & 6 \\ 1 & 4 & 10 \\ 6 & 9 & 3 \end{bmatrix}$$

Further, matrices are usually named capital letters, the most common being A or M in this course. The matrix is referred to by this letter, and we can refer to certain elements within the material based on its **index**. The index refers to the coordinate of the entry in the matrix, and is denoted also by **row, column** with the matrix letter as a lower case letter. For example, from our square matrix above, the 4 in the middle would be referred to as $a_{2,2}$ if it were in matrix A because it is in the 2nd row and 2nd column. The 9 below it would be $a_{3,2}$ because it is in the 3rd row and 2nd column.

Also, important vocabulary includes the **diagonal** of a matrix, which is just as it sounds, the entries starting at the top left entry, to the one diagonal to that, and so on.

Common matrices

Zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(any matrix will all 0 entries)

Column matrix

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(any $m \times 1$ matrix)

Row matrix

$$\begin{bmatrix} 1 & 5 & 6 \end{bmatrix}$$

(any $1 \times m$ matrix)

Diagonal matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

(any matrix with all non-zero entries diagonal)

Identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(any square matrix will 1's along diagonal)

Upper triangular matrix

$$\begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$$

(any matrix with all non-zero entries on/above diagonal)

Lower triangular matrix

$$\begin{bmatrix} 1 & 0 \\ 3 & 9 \end{bmatrix}$$

(any matrix with all non-zero entries on/below diagonal)

Matrix Transpose

You can calculate the transpose of a **square or non-square matrix**. The main point of a transpose is to switch the rows and columns of a matrix. This is done essentially by reflecting entries over the diagonal of the matrix. This means that for a $k \times n$ matrix A, the transpose of the matrix, denoted A^T, is $n \times k$, since we are switching the rows and columns.

The transpose operation is **involutive**, which means that performing the operation twice returns it to the original matrix, $(A^T)^T = A$.

Here are some examples:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Symmetric matrix

$$\begin{bmatrix} 1 & 5 \\ 5 & -1 \end{bmatrix}$$

(any square matrix where $A^T = A$)

Skew symmetric matrix

$$\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

(any square matrix where $A^T = -A$)
(therefore all diagonal entries must $= 0$)